WEEKLY TEST MEDICAL PLUS -03 TEST - 08 RAJPUR
SOLUTION Date 08-09-2019

[PHYSICS]

1.

Because the body is revolving in a circle with constant speed, hence acceleration acting on it is exactly perpendicular to direction of its motion, *i.e.*, the body possesses normal acceleration.

2.

Because the particle moving in a circle describes equal angles in equal times, hence both ω and r are constant. Thus, magnitude of velocity vector remains constant but the direction changes from point to point.

3.

$$v = R_e\omega = R_e \times \frac{2\pi}{T}$$

$$= 4000\,\text{miles} \times \frac{2\pi}{24\,\text{hr}} \simeq 1000\,\text{mile/hr.}$$

4.

$$\text{Acceleration} = \omega^2 r = (2\pi f)^2 r = 4\pi^2 f^2 r$$
$$= 4\pi^2 \times 1 \times (2\times 10^4) = 8\times 10^5\,\text{m/s}^2.$$

5.

Displacement, velocity and acceleration change continuously with respect to time because of change in direction.

6.

Angular speed of the particle, *i.e.*, rate of change of angular displacement of the particle remains constant.

7.

The required retardation is given by:

$$a = \frac{v^2}{2x} = \frac{20\times 20}{2\times 20} = 10\text{ m s}^{-2}$$

The centripetal acceleration required to describe a circle of radius 20 m is,

$$\frac{v^2}{R} = \frac{20\times 20}{20} = 20\text{ m s}^{-2}$$

Thus, it is better to apply the brakes.

8.

In circular motion, centripetal force acting on the body is always perpendicular to the velocity vector or displacement vector. Hence, work done $(= \vec{F} \cdot \vec{d})$ is always zero whatever may be the displacement along the circular path.

9.

As, $T_1 = T_2$

Hence, $\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$ or $\frac{v_1}{v_2} = \frac{r_1}{r_2}$

$$\frac{F_1}{F_2} = \frac{mv_1^2}{r_1} \times \frac{r_2}{mv_2^2} = \left(\frac{v_1}{v_2}\right)^2 \times \frac{r_2}{r_1} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{r_2}{r_1} = \frac{r_1}{r_2}$$

10.

Since, water does not fall down, therefore, the velocity of revolution should be just sufficient to provide centripetal acceleration at the top of vertical circle. So,

$$v = \sqrt{gr} = \sqrt{10 \times 1.6} = 4 \text{ m/s}$$

11.

Because the particle is moving in a circle with uniform speed, hence kinetic energy $\left(= \frac{1}{2}mv^2\right)$ will remain constant. Acceleration, velocity and displacement will change from point to point due to change in direction.

12.

$v = \sqrt{5gr} = \sqrt{5 \times 9.8 \times 4} = \sqrt{196} = 14$ m/s.

13.

Acceleration of a point at the tip of the blade

$= \text{centripetal acceleration} = \omega^2 R = (2\pi f)^2 R$

$$= \left(2 \times \frac{22}{7} \times \frac{1200}{60}\right)^2 \times \frac{30}{100} = 4740 \text{ m/sec}^2$$

14.

Centripetal force required for negotiating the curve $= \frac{Mv^2}{R}$.

When velocity is doubled, centripetal force required is quadrupled, *i.e.*, tendency to overturn is also quadrupled.

15.

Velocity at the top is $\sqrt{gr}$ and that at the bottom is $\sqrt{5gr}$. Hence, required difference in kinetic energy

$$= \frac{1}{2} M[5gr - gr] = 2Mgr$$

$$= 2 \times 10 \times 1 \times 1 = 20 \text{ J}.$$

16.

Centripetal force = force of friction

$$\frac{Mv^2}{r} = \mu \times \text{reactional force}$$

or $\frac{Mv^2}{r} = \mu Mg$ or $v = \sqrt{\mu rg}$.

17.

To cross the bridge without leaving the ground, at the highest point of the bridge,

$$\frac{Mv^2}{R} = Mg \quad \text{or} \quad v = \sqrt{Rg}\,.$$

18.

Angular momentum, $L = r \times p = r \times m \times v$

or $$v = \frac{L}{mr} \qquad \ldots\text{(i)}$$

Now, as centripetal force, $F_c = \frac{mv^2}{r}$ ...(ii)

Substituting the value of v from eqn. (i) in eqn. (ii), we get;

$$F_c = \frac{m}{r}\left[\frac{L}{mr}\right]^2 = \frac{L^2}{mr^3}\,.$$

19.

Length of the path,

$$314 = \frac{2\pi r}{4} \quad \text{or} \quad r = 200 \text{ m}$$

$$\therefore \quad F = \frac{mv^2}{r} = \frac{1500 \times (20)^2}{200} = 3000 \text{ N.}$$

20.

21.

$$M = 4 \text{ kg}, \quad f = \frac{120}{60} \text{ per sec}, r = 2 \text{ m}$$

$$K = \frac{1}{2} Mv^2 = \frac{1}{2} Mr^2\omega^2$$

$$= \frac{1}{2} Mr^2 (2\pi f)^2$$

$$= \frac{1}{2} \times 4 \times 4 \times 4\pi^2 \times 4 = 1263 \text{ J}\,.$$

22.

Given that masses and time periods of two bodies are same,

$$F = m\omega^2 R = m\left(\frac{2\pi}{T}\right)^2 R$$

As m and T are same for two bodies, hence

$$\frac{F_1}{F_2} = \frac{R_1}{R_2}\,.$$

23.

During upward motion,

$$T_1 = m(g + a) = 1(9.8 + 4.9) = 14.7 \text{ N}$$

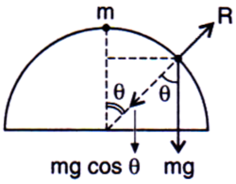
During downward motion,

$$T_2 = m(g - a) = 1(9.8 - 4.9) = 4.9 \text{ N}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{14.7}{4.9} = 3\,.$$

24.

If the radius vector makes an angle θ with the vertical, then



$$mg\cos\theta - R = \frac{mv^2}{r}$$

When the body leaves the surface,

$$R = 0$$

$\therefore$ $$mg\cos\theta = \frac{mv^2}{r}$$

or $$\cos\theta = \frac{v^2}{rg} = \frac{(5)^2}{5\times 10} = \frac{1}{2}$$

$\therefore$ $$\theta = 60°.$$

25.

$$v_{max.} = \sqrt{\mu rg} = \sqrt{0.3\times 10\times 300}$$

$$= 30 \text{ m/sec} = 30\times\frac{18}{5} = 108 \text{ km/hr}$$

26.

$$v = 4.9 \text{ m/sec}, r = 4 \text{ m}, \mu = \frac{v^2}{rg} = \frac{(4.9)^2}{4\times 9.8} = 0.61.$$

27.

Rate of change of speed,

$$\frac{dv}{dt} = \text{tangential acceleration}$$

$$= \frac{\text{tangential force}}{\text{mass}} = \frac{mg\sin 30°}{m}$$

$$= g\sin 30° = 10\times 1/2 = 5 \text{ m/s}^2.$$

28.
29.
30.

$$M = 6\times 10^{24} \text{ kg}, \omega = 2\times 10^{-7} \text{ rad/sec}$$

$$r = 1.5\times 10^8 \text{ km} = 1.5\times 10^{11} \text{ m}$$

Force exerted on the earth

$$= m\omega^2 r = (6\times 10^{24})\times(2\times 10^{-7})^2\times(1.5\times 10^{11})$$

$$= 36\times 10^{21} \text{ N}.$$

31.
32.
33.
34.

Difference in kinetic energy

$$= 1/2m[(\sqrt{5gr})^2 - (\sqrt{gr})^2]$$

$$= 2\,mgr = 2\times 1\times 10\times 1 = 20 \text{ J}.$$

35.

When the string is released, tension in the string becomes zero and the stone flies along the tangent to the circle because its velocity is directed along the tangent.

36.

Angular momentum is directed along a line perpendicular to the plane of rotation.

We know that $\vec{L} = \vec{r} \times \vec{p}$

According to definition of cross product, direction of $\vec{L}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{p}$.

37.

Tangential acceleration, $a_t = r\alpha = 4 \text{ m/s}^2$

Radial acceleration,

$$a_r = \omega^2 r = \frac{v^2}{r} = \frac{60 \times 60}{1200} = 3 \text{ m/s}^2$$

Hence, resultant acceleration of the car

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{4^2 + 3^2} = 5 \text{ m/s}^2.$$

38.

The block falls through a vertical height equal to the radius of the circle.

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ ms}^{-1}.$$

39.

Tension at the top $= \frac{mv^2}{r} - mg$

Tension at the bottom $= \frac{mv^2}{r} + mg$

Tension in horizontal position $= \frac{mv^2}{r}$

Tension at the top $= m\left[\frac{v^2}{r} - g\right] = 1 \times \left[\frac{16}{1} - 10\right]$

$= 6$ N.

40.

If v is the velocity at the mean position, then

$$\frac{1}{2} mv^2 = mgl \quad \text{or} \quad \frac{mv^2}{l} = 2mg$$

Tension at the lowest point,

$$T - mg = \text{centripetal force}$$

or $T - mg = 2mg \quad \therefore T = 3mg$.

41.

Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8 \text{ m/s}^2$

Tangential acceleration, $a_t = 2 \text{ m/s}^2$

$\therefore$ Resultant acceleration $a = \sqrt{a_t^2 + a_c^2}$

$$= \sqrt{(1.8)^2 + (2)^2} = 2.7 \text{ m/s}^2$$

42.

Radial acceleration, $a_r = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8 \text{ m s}^{-2}$

Tangential acceleration, $a_t = 2 \text{ m/sec}^2$

Resultant acceleration,

$$a^2 = a_r^2 + a_t^2 + 2a_r a_t \cos\theta$$

Here, $\theta = 90°$

$\therefore$ $a^2 = (1.8)^2 + (2)^2 + 0 = 3.24 + 4 = 7.24$

$\therefore$ $a = \sqrt{7.24} = 2.7 \text{ ms}^{-2}$

43.

Tension at the top of circular motion in a vertical plane

$$= \frac{Mv^2}{R} - Mg$$

$$T = \frac{1 \times 16}{1} - 1 \times 10 = 6 \text{ N.}$$

44.

The outer rail of the curved railway track is raised above the inner one, to provide the necessary centripetal force for circular motion along the curved railway track.

45.

$$\omega^2 r = 4\pi^2 n^2 r = 4\pi^2 \left(\frac{1200}{60}\right)^3 \times 30 = 4740 \, m/s^2$$